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Flow of Conducting Fluid on solid Core Surrounded by Porous Cylindrical Region in Presence of Transverse Magnetic Field

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Abstract

The steady flow of an electrically conducting, viscous and incompressible fluid flow through / past a solid core surrounded by cylindrical porous medium is considered in the presence of the transverse magnetic field. The modified Brinkman and Stokes equations are used to describe the fluid flow in porous and non-porous regions respectively. The exact solution is obtained in terms of modified Bessel's function. The matching boundary conditions are used at the interface of the two regions along with the no-slip condition on the surface of the solid core. Further, uniform velocity away from the fluid surface is considered. The effect of magnetic field and porous parameter on the fluid flow is presented for both porous and non-porous regions. From the obtained result it is noticed that increase in magnetic field strength, the flow is suppressed and fluid flow through porous region is observed. Further, increase in porous

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parameter, offered a resistance to the fluid flow through porous medium thereby fluid flow past porous region is noticed.

Keywords: Incompressible, Brinkman equation, Stokes equation, matching boundary, porous parameter.

1. Introduction

The flows of the fluids through / past a porous media are of principle interest because of its natural occurrence and its importance in industrial, geophysical and bio-medical applications. In chemical industries it has been used to achieve an effective mixing process, filtration, purification process, oil recovery. In nuclear industries, porous medium is used for effective insulation and for emergency cooling of nuclear reactors. To study underground water

resources, seepage of water in river beds, we need to known the concept of fluid flow through porous media. Also, it helps in bio-medical problem to understand the transport process of lungs and kidneys.

In the literature, studies are found on the fluid flow past a sphere or cylinder in porous media using Darcy's or Brinkman equation under different boundary conditions. Beaver's and Joseph [1], Saffman [2] Raja Sekhar and Amaranath [3], they have used Darcy's equation to describe the flow field. The problem of Stokes flow past porous bodies have been studied by Masliyah et al [4], Qin and Kaloni [5], Barman[6], Pop and Ingham [7], Anindita Bhattacharya and Raja Sekhar [8], Pop et al [9] using Brinkman model in porous region to describe the motion.

The study of hydrodynamic flows in presence of magnetic field has attracted many authors due to vast applications in astrophysical, geophysical and industrial fields. Many practical problems need a mechanism to control the motion of the fluid past solid bodies with Magnetohydrodynamics (MHD) effects. The study of magnetohydrodynamic flows of electrically conducting fluids in electric and magnetic fields is of considerable interest in modern metallurgical and metal working processes. This has led to considerable interest in the study of boundary layer flows subjected to an externally applied magnetic field.

Anjali Devi and Raghavachar [10] studied the horizontal flow of a vertically stratified, electrically conducting fluid past a non conducting sphere in the presence of uniform magnetic field for non-diffusive medium. Kyrlidis et al [11] presented the study of conducting fluid past axi-symmetric bodies in the presence of magnetic field in the limit of small inertial and magnetic Reynolds numbers. The objective was to control the particle settling in metallic systems. Chandran et al [12] have been analyzed the effect of magnetic field on the flow heat transfer past a continuously moving porous plate in a stationary fluid and using similarity transformation method, the governing boundary layer equations are reduced to a set of non-linear ordinary differential equations. Numerical solutions for the velocity and temperature functions have been found by shooting method using Runge-Kutta algorithm. The steady, viscous, electrically conducting fluid flow around a circular cylinder in the presence of magnetic field applied parallel to the main flow was investigated by Raghava Rao and Sekhar [13]. Finite difference method was used to solve the non-linear Navier-Stokes equation. Jayalakshmamma et al [14] presented a creeping flow past a composite sphere in presence of magnetic field. matching boundary conditions are applied at the interface of the fluid and porous media. Pal and Talukdar [15] analyzed an investigation on the unsteady flow of a laminar two-dimensional oscillatory flow of an incompressible electrically conducting viscous fluid between two non-conducting parallel plane surfaces in the presence of suction / injection. An analytical solution was obtained by adopting regular perturbation technique.

The influence of magnetic field to control the flow during the relative motion of a body or fluid is the main objective of this article. Therefore in this paper, the creeping flow of a steady, incompressible, viscous, electrically conducting fluid flow past a solid cylinder embedded in a cylindrical porous medium is presented in the presence of transverse magnetic field. The analytical method is given to find an exact solution for the considered fluid flow. The matching boundary conditions for velocity and stress across the interface is considered. Further, it is assumed that the induced magnetic field is negligible compared to the applied magnetic field.

2. Mathematical formulation

The steady flow of viscous, incompressible and conducting fluid through and past a porous cylindrical region of radius *b* comprising a solid cylindrical core of radius *a*, is investigated in the presence of transverse magnetic field. It is assumed that the induced magnetic field is negligible, as the magnetic Reynolds number is small. Also the flow domain has been divided into two regions as non-porous (fluid region) and porous region. The constitutive equations which describe the flow of a conducting fluid in non-porous region under the assumption made are modified Stokes's equation with the equation of continuity and are given by:

Equation of continuity

$$\nabla \cdot \vec{q}_1 = 0, \tag{1}$$

and modified Stokes equation

$$\nabla p_1 = \mu \nabla^2 \vec{q}_1 + \mu_h^2 \sigma_e \left(\vec{q}_1 \times \vec{H} \right) \times \vec{H}, \tag{2}$$

where $\vec{q}_1 = (u_1, v_1, w_1)$ is the velocity in the non-porous region, μ is the viscosity of the fluid, μ_h is the magnetic permeability, σ_e is the electrical conductivity, which is very small so that the induced magnetic field is negligible, $\vec{H}$ is the uniform magnetic field and p_1 is the hydrostatic pressure of the fluid region. Here equation (2) is said to be modified Stokes equation as it consist of Lorentz force due to applied Magnetic field, along with the viscous term on the right hand side of the equation.

The flow in the porous region $a < r \leq b$ is governed by the modified Brinkman equation along with equation of continuity, given by:

$$\nabla \cdot \vec{q}_2 = 0, \tag{3}$$

$$\nabla p_2 = \bar{\mu} \nabla^2 \vec{q}_2 - \frac{\mu}{k} \vec{q}_2 + \mu_h^2 \sigma_e \left(\vec{q}_2 \times \vec{H} \right) \times \vec{H}, \tag{4}$$

where, $\vec{q}_2 = (u_2, v_2, w_2)$ is the velocity in the porous region, $\bar{\mu}$ is the Brinkman viscosity, p_2 the hydrostatic pressure of the porous region and k the permeability of the porous region.

In this study, the cylindrical polar co-ordinates are used. Thus, for an axi-symmetric, two dimensional flow in a cylindrical co-ordinate system (r, θ, z) with the origin at the center of the cylinder and the axis $\theta = 0$ is chosen along the direction of the uniform velocity u_∞ far from the non-porous region. Also due to axi-symmetry we have $\frac{\partial}{\partial z} = 0$. The flow characteristics of the problem are described by equations (1) to (4) can be analyzed in terms of non-dimensional parameters pertaining to the flow processes. In view of this, the following dimensionless similarity variables are introduced:

$$r^* = \frac{r}{a}, \quad \vec{q}_1^* = \frac{\vec{q}_1}{u_\infty}, \quad \vec{q}_2^* = \frac{\vec{q}_2}{u_\infty}, \quad \vec{H}_1^* = \frac{\vec{H}_1}{H_0}, \quad p_1^* = \frac{a p_1}{\mu u_\infty}, \quad p_2^* = \frac{a p_2}{\mu u_\infty}, \tag{5}$$

where H_0 is the applied constant magnetic field.

After non-dimensionalising the governing equations (1) to (4) using the non-dimensional variables as defined in equation (5) for cylindrical polar co-ordinate system in fluid region, we get:

$$\frac{\partial}{\partial r}(r u_1) + \frac{\partial v_1}{\partial \theta} = 0, \tag{6}$$

$$-\frac{\partial p_1}{\partial r} = M^2 u_1 - \left(\frac{\partial^2 u_1}{\partial r^2} + \frac{1}{r}\frac{\partial u_1}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u_1}{\partial \theta^2} - \frac{u_1}{r^2} - \frac{2}{r^2}\frac{\partial v_1}{\partial \theta} \right), \tag{7}$$

$$-\frac{1}{r}\frac{\partial p_1}{\partial \theta} = M^2 v_1 - \left(\frac{\partial^2 v_1}{\partial r^2} + \frac{1}{r}\frac{\partial v_1}{\partial r} + \frac{1}{r^2}\frac{\partial^2 v_1}{\partial \theta^2} - \frac{v_1}{r^2} + \frac{2}{r^2}\frac{\partial u_1}{\partial \theta} \right), \quad (8)$$

here $(u_1, v_1, 0)$ represents the velocity of the fluid in the fluid region, p_1 the pressure in fluid region. M Hartmann number (as defined above). Similarly, the non-dimensionalised governing equation for the porous region takes the form:

$$\frac{\partial}{\partial r}(r u_2) + \frac{\partial v_2}{\partial \theta} = 0, \quad (9)$$

$$-\frac{\partial p_2}{\partial r} = S^2 u_2 - \left(\frac{\partial^2 u_2}{\partial r^2} + \frac{1}{r}\frac{\partial u_2}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u_2}{\partial \theta^2} - \frac{u_2}{r^2} - \frac{2}{r^2}\frac{\partial v_2}{\partial \theta} \right), \quad (10)$$

$$-\frac{1}{r}\frac{\partial p_2}{\partial \theta} = S^2 v_2 - \left(\frac{\partial^2 v_2}{\partial r^2} + \frac{1}{r}\frac{\partial v_2}{\partial r} + \frac{1}{r^2}\frac{\partial^2 v_2}{\partial \theta^2} - \frac{v_2}{r^2} + \frac{2}{r^2}\frac{\partial u_2}{\partial \theta} \right), \quad (11)$$

here $(u_2, v_2, 0)$ components of velocity in normal and tangential direction in porous medium, p_2 the static pressure in porous region and $S^2 = M^2 + \sigma^2$ in which $\sigma = \frac{a}{\sqrt{k}}$ is the porous parameter and k is the permeability of the fluid.

As the flow is axi-symmetric and two dimensional, the stream function $\psi_i(r,\theta)$(where $i = 1, 2$ correspondingly for fluid and porous regions) is introduce, which satisfies the equation of continuity in cylindrical polar co-ordinate system for both non-porous and porous regions respectively

$$u_i = \frac{1}{r}\frac{\partial \psi_i}{\partial \theta}; \quad v_i = -\frac{\partial \psi_i}{\partial r}, \quad (12)$$

here u_i is the normal component of velocity and v_i is the tangential velocity. By eliminating the pressure term from equations (6), and (7) of non-porous region and equations (9), and (10) of porous region by cross differentiation we get a fourth order linear partial differential equation in terms of corresponding stream function as:

$$\nabla^4\psi_1 - M^2\nabla^2\psi_1 = 0, \quad b \le r < \infty, \tag{13}$$

$$\nabla^4\psi_2 - S^2\nabla^2\psi_2 = 0, \quad a \le r < b, \tag{14}$$

where $\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}$ is Laplacian operator in cylindrical polar co-ordinate system.

In the present problem, physically realistic and mathematically consistent boundary conditions are used. These are the no-slip condition on the solid cylindrical core, continuity of velocities and stresses across the interface of two regions and uniform velocity far away from the fluid region. No-slip conditions on the surface of the solid cylindrical core are:

$$u_2(a,\theta) = 0, \quad 0 \le \theta \le 2\pi, \tag{15}$$

$$v_2(a,\theta) = 0, \quad 0 \le \theta \le 2\pi. \tag{16}$$

The interfacial conditions, continuity of normal and tangential velocity components, continuity of normal and tangential stress components at the interface of the porous and fluid region are given by:

$$u_2(b,\theta) = u_1(b,\theta), \quad 0 \le \theta \le 2\pi, \tag{17}$$

$$v_2(b,\theta) = v_1(b,\theta), \quad 0 \le \theta \le 2\pi, \tag{18}$$

$$\tau_{r\theta(2)}(b,\theta) = \tau_{r\theta(1)}(b,\theta), \quad 0 \le \theta \le 2\pi, \tag{19}$$

$$\tau_{rr(2)}(b,\theta) = \tau_{rr(1)}(b,\theta), \quad 0 \le \theta \le 2\pi, \tag{20}$$

where $\tau_{r\theta(i)}$ and $\tau_{rr(i)}$ are the dimensionless tangential and normal components of stress tensors, written in cylindrical co-ordinate as:

$$\tau_{r\theta(i)} = \frac{1}{r}\frac{\partial u_i}{\partial \theta} + \frac{\partial v_i}{\partial r} - \frac{v_i}{r}, \tag{21}$$

$$\tau_{rr(i)} = -p_i + 2\frac{\partial u_i}{\partial r}. \tag{22}$$

The continuity of the normal stress at the interface of the two regions from the boundary condition (20), shows that the continuity of pressure across the interface, since the viscosity of the fluid is equal to the Brinkman viscosity $\bar{\mu} = \mu$. Therefore, equation (20) reduces to:

$$p_2(b,\theta) = p_1(b,\theta), \quad 0 \le \theta \le 2\pi. \tag{23}$$

Also, the uniform velocity far away from the fluid cylindrical region is given by:

$$\psi_1(r,\theta) \sim r\sin\theta \ \text{ as } \ r \to \infty. \tag{24}$$

3. Method of solution

The boundary condition of uniform velocity far away from the porous cylindrical region leads to find the solution for the fourth order partial differential equations of equations (13) and (14) by similarity solution method as

$$\psi_i(r,\theta) = f_i(r)\sin\theta. \tag{25}$$

Substituting equation (25) in equations (13) and (14) in respective regions, we obtain the ordinary differential equation of order four in $f_i(r)$ as

$$f_i^{iv} + \frac{2}{r} f_i''' - \frac{3}{r^2} f_i'' + \frac{3}{r^3} f_i' - \frac{3}{r^4} f_i - J_i^2\left[f_i'' + \frac{1}{r} f_i' - \frac{1}{r^2} f_i\right] = 0, \tag{26}$$

where $J_i^2 = \begin{cases} M^2 & i = 1 \\ S^2 & i = 2 \end{cases}.$

The corresponding boundary conditions are:

No-slip condition at the surface of the solid cylinder is given by

$$f_2(a)=0, \tag{27}$$

$$f_2'(a)=0. \tag{28}$$

The continuity of the velocity and stresses at the interface of the porous and fluid region is given by

$$f_2(b)= f_1(b), \tag{29}$$

$$f_2'(b)= f_1'(b), \tag{30}$$

$$f_2''(b)= f_1''(b), \tag{31}$$

$$f_2'''(b)-\sigma^2 f_2' = f_1'''(b). \tag{32}$$

Further, the uniform velocity far away from the fluid region is reduces to:

$$f_1(r)\sim r \text{ as } r\to\infty. \tag{33}$$

The forth order ordinary differential equation (26), is converted into second order by taking the substitution

$$g_i(r)= f_i''(r)+\frac{1}{r} f_i'(r)-\frac{1}{r^2} f_i(r), \tag{34}$$

The equation (26) reduces to

$$g_i''(r)+\frac{1}{r}g_i'(r)-\left[J_i^2+\frac{1}{r^2}\right]g_i(r)=0. \tag{35}$$

Equation (35) represents the modified Bessel's differential equation of order one, whose general solution is of the form

$$g_i(r)=C_iK_1(J_ir)+D_iI_1(J_ir), \tag{36}$$

where C_i and D_i are the arbitrary constants, and substituting equation (36) in equation (34) we get

$$f_i''(r)+\frac{1}{r} f_i'(r)-\frac{1}{r^2} f_i(r)=C_iK_1(J_i r)+D_iI_1(J_i r). \tag{37}$$

Here, equation (37) is an ordinary second order differential equation with variable co-efficient which can be solved by the method of variation of parameter and the obtained complete solution is

$$f_i(r)=\frac{A_i}{r}+B_i r+C_iK_1(J_i r)+D_iI_1(J_i r). \qquad 38)$$

Therefore, the solutions of equation (26) for non-porous and porous medium are given by:

$$f_1(r)=\frac{A_1}{r}+B_1 r+C_1 K_1(M\,r)+D_1 I_1(M\,r) \qquad b\le r<\infty, \tag{39}$$

$$f_2(r)=\frac{A_2}{r}+B_2 r+C_2 K_1(S\,r)+D_2 I_1(S\,r) \qquad a<r<b, \tag{40}$$

where $A_1, B_1, C_1, D_1, A_2, B_2, C_2$ and D_2 are arbitrary constants. In the fluid region as $r\to\infty$, then $I_1(Mr)\to\infty$. Therefore the solution is valid for $D_1=0$ and also due to the boundary condition for uniform velocity far away from the medium, from equation (33) we get $B_1=1$. Thus equation (39) reduces to:

$$f_1(r)=\frac{A_1}{r}+r+C_1 K_1(M\,r) \qquad b<r<\infty. \tag{41}$$

Hence the stream function in both the regions takes the form

$$\psi_1(r,\theta)=\left(\frac{A_1}{r}+r+C_1 K_1(M\,r)\right)\sin\theta \qquad b\le r<\infty, \tag{42}$$

$$\psi_2(r,\theta)=\left(\frac{A_2}{r}+B_2 r+C_2 K_1(S\,r)+D_2 I_1(S\,r)\right)\sin\theta \quad a<r\le b. \tag{43}$$

The arbitrary constants present in the equations (42) and (43) are evaluated using the boundary conditions and are given in the appendix. Further, the expression for normal and tangential component of velocities for both porous and non-porous regions can be obtained in terms of stream function from equation (12).

4. Results and discussion

The viscous flow of steady incompressible electrically conducting fluid past a solid cylinder embedded in a cylindrical porous medium has been investigated in the presence of uniform magnetic field, applied in the transverse direction of the fluid motion. The modified Stokes and Brinkman equations are used to illustrate the flow behavior in fluid and porous regions respectively. The induced magnetic field is assumed to be negligible, since the conductivity and magnetic Reynolds number are very small. Analytical solution is obtained by similarity solution method. In this method the partial differential equation of the physical configuration are transformed to ordinary differential equations. These ordinary differential equations are converted into modified Bessel's equations using a special transformation, whose solution is obtained in terms of modified Bessel function of order one. The continuity of velocity, tangential and normal stress are used as the interface boundary conditions between fluid and porous regions. Also, the no-slip condition at the surface of the solid cylinder and uniform velocity away from the porous cylinder are considered. Finally, the expression for stream function is obtained as a function of r and dimensionless parameters.

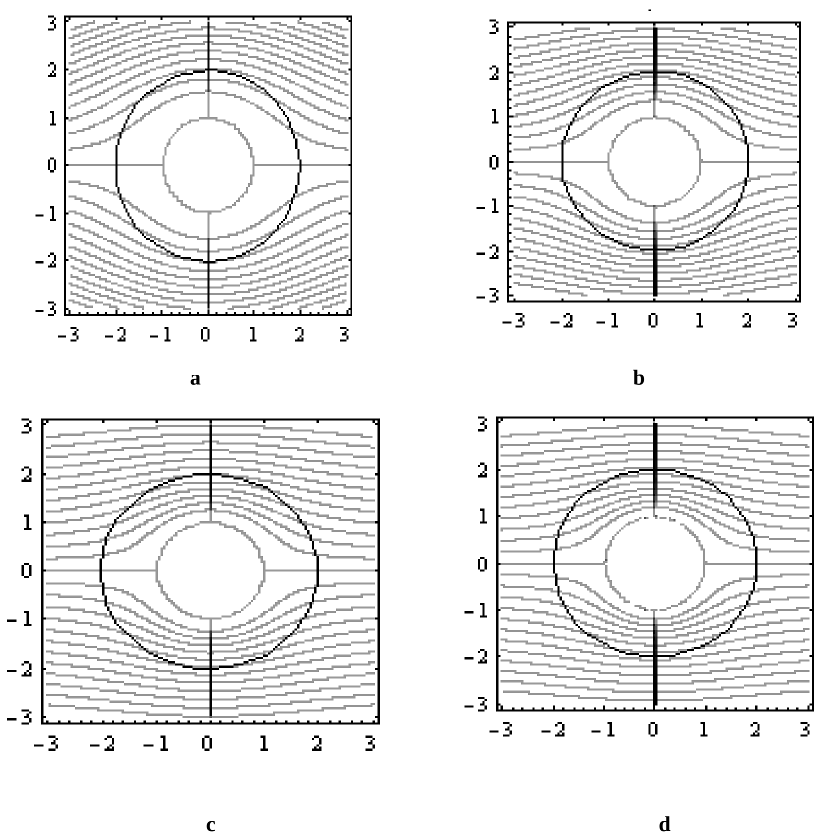

Fig 1: Streamlines for fixed $\sigma = 0.001$ and different value of M (a) $M = 0.1$ (b) $M = 2$ (c) $M = 5$ (d) $M = 10$

The effect of both magnetic field M and porous parameter σ on the flow patterns are discussed through the streamlines. First, the effect of magnetic field for negligible value of σ is studied. For small magnetic field strength $M = 0.1$, it is noticed that the streamlines are free and are away from the solid cylinder. By increasing the magnetic field strength to $M = 2$, the streamlines are moved towards the cylindrical core and more amount of fluid moves inside the porous region. Further increase in Hartmann number $(M = 5, 10)$, the streamlines are meandering near the core and the observations are shown in Figs. 1 (a) to (d).

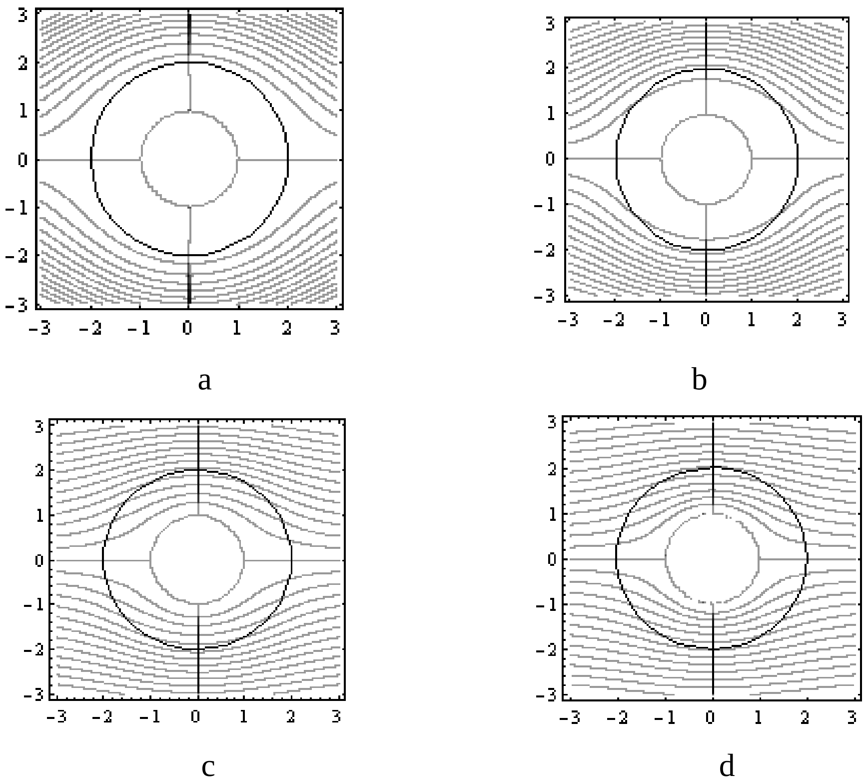

Fig 2: Streamlines for fixed $\sigma = 5$ and different value of M (a) $M = 0.1$ (b) $M = 2$ (c) $M = 5$ (d) $M = 10$

For an increase in $\sigma = 5$ by fixing Hartmann number $(M = 0.001)$ at negligible value, from Fig. 2(a), it is noticed that the fluid is flowing past the porous cylinder rather than passing through it. This can be attributed to the lower permeability of the porous medium. For this σ, when the magnetic field strength is increased the fluid starts to move inside the porous region. As a result the streamlines are moving closer to the solid surface of a cylinder and the same is depicted in Figs. 2 (b) to (d).

On the other hand, for negligible Hartmann number $(M = 0.001)$ when the porous parameter is increased, it is found that the behavior of the fluid flow is opposite. For small porous parameter $(\sigma = 1)$ value, free flow is observed in porous cylindrical region. However, as the value of σ is increased for the same Hartmann number, the flow behavior has been changed completely. As the value of porous parameter increases, the fluid flow past the porous cylinder, this is similar to the flow of viscous fluid and is shown in Figs. 3 (a) to (d).

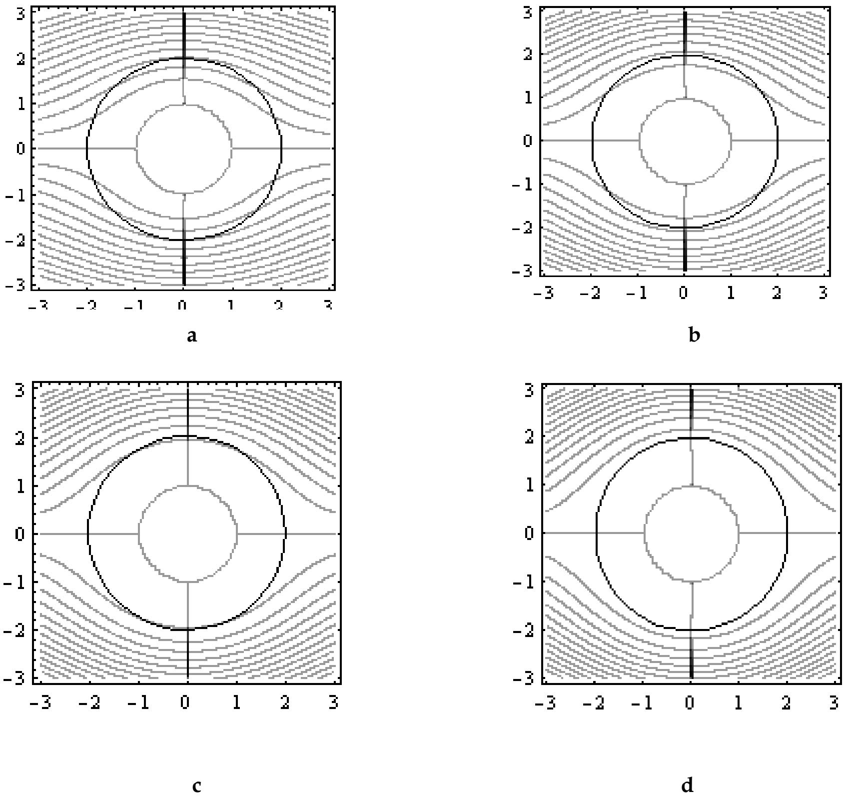

Fig 3: Streamlines for fixed $M = 0.001$ and different value of σ, (a) $\sigma = 1$ (b) $\sigma = 2$ (c) $\sigma = 3$ (d) $\sigma = 5$

5. Conclusions

The present study projects on certain practical applications such as metallergy and metal processing, lubrication and in nuclear reactors, where an additional force such as magnetic field is applied to control the fluid flow. In the view of these applications, this papers presents the analytical solution for steady flow of an incompressible viscous and electrically conducting fluid past a solid cylinder placed in a cylindrical porous medium, in the presence of transverse magnetic field. The influence of Hartman number and porous parameter are discussed on the streamline patterns. From the graph, the meandering of streamlines near the surface of the solid cylinder is noticed for the increase in magnetic field strength with fixed or negligible porous parameter. This shows that the fluid flow is effectively controlled by the magnetic field as a result more amount of fluid flows through the porous region / on the surface of the solid core.

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Appendix

$$C2=\frac{n_7n_{12}-n_{10}n_9}{n_7n_{11}-n_{10}n_8};\ D2=\frac{1}{n_7}\left(n_9-n_8C2\right);$$

$$B2=\frac{1}{m_{17}}\left(m_{17}-m_{19}C2-m_{18}D2\right);$$

$$A2=\frac{1}{m_{12}}\left(m_{16}-m_{15}C2-m_{14}D2-m_{13}B2\right);$$

$$C1=\frac{1}{m_1}\left(2b^2-m_3C2-m_2D2-2b^2B2\right);$$

$$A1=-b^2-l_3C2-l_2D2+b^2B2+A2-l_1C1;n_1=a^2m_{12}-m_{13};$$

$$n_2=m_{12}l_{15}-m_{14};n_3=m_{12}l_{16}-m_{15};n4=a^2m12+m13;n_5=m_{12}l_{17}+m_{14}$$

$$;n6=m12l18+m15;\ n_7=m_{17}n_2-m_{18}n_1;$$

$$n_8=m_{17}n_3-m_{19}n_1;n_9=-m_{17}\left(m_{16}+n_1\right);\ n_{10}=m_{17}n_5-m_{18}n_4;$$

$$n_{11}=m_{17}n_6-m_{19}n_4;\ n_{12}=-m_{17}\left(m_{16}-n_4\right);\ m_1=l_4-l_1;$$

$$m_2=l_5-l_2;m_3=l_6-l_3;$$

$$m_4=l_7+6l_1;m_5=l_8-6;m_6=l_9-6b^2;m_7=l_{10}+6l_2;m_8=l_{11}+6l_3;$$

$$m_9=l_{12}-2l_1;m_{10}=l_3-2l_2;$$

$$m_{11}=l_{14}-2l_3;m_{12}=m_1m_5;m_{13}=m_1m_6-2b^2m_4;m14=m1m7-m2m4;$$

$$m15=m1m8-m3m4;m16=-2b^2\left(3m1+m4\right);m17=2b^2\left(m1-m9\right);$$

$$l8=6-\sigma^2b^2\ \ m18=m1m10-m2m9;m19=m1m11-m3m9;$$

$$l1=b\,K_1\left(M\,b\right);\ l2=-bI_1\left(S\,b\right);\ l3=-b\,K_1\left(S\,b\right);$$

$$l4=-b^2\left[\frac{K_1\left(M\,b\right)}{b}-M\,K_2\left(M\,b\right)\right];l5=b^2\left[\frac{I_1\left(S\,b\right)}{b}+SI_2\left(S\,b\right)\right];$$

$$l6=b^2\left[\frac{K_1\left(S\,b\right)}{b}-S\,K_2\left(S\,b\right)\right];$$

$$l7=b^4\left[-\frac{3M}{b^2}K_2\left(Mb\right)+\frac{6M^2}{b}K_3\left(Mb\right)-M^3\,K_4\left(M\,b\right)\right];\ l9=\sigma^2b^4;$$

$$l10 = b^4\left[\sigma^2\left(\frac{I_1(Sb)}{b} + SI_2(Sb)\right) - \left(\frac{3S}{b^2} I_2(Sb) + \frac{6S^2}{b} I_3(Sb) + S^3 I_4(S\,b)\right)\right]$$

;

$$l11 = b^4\left[\sigma^2\left(\frac{K_1(Sb)}{b} - SK_2(Sb)\right) - \left(-\frac{3S}{b^2} K_2(Sb) + \frac{6S^2}{b} K_3(Sb) - S^3 K_4(S\,b)\right)\right]$$

;

$$l12 = b^3\left[-\frac{3M}{b} K_2(M\,b) + M^2 K_3(M\,b)\right];$$

$$l13 = -b^3\left[\frac{3S}{b} I_2(S\,b) + S^2 I_3(M\,b)\right];$$

$$l12 = -b^3\left[-\frac{3S}{b} K_2(S\,b) + S^2 K_3(S\,b)\right]; l15 = aI_1(S\,a);$$

$$l16 = a\,K_1(S\,a); l17 = a^2\left[\frac{I_1(S\,a)}{a} + SI_2(S\,a)\right];$$

$$l18 = a^2\left[\frac{K_1(S\,a)}{a} - S\,K_2(S\,a)\right];$$